How does the tension affect the frequency of a standing wave?

Rationale:

Waves occur naturally, showing how energy is used for different functions. Waves are prevalent in our daily lives since they are used in radio and television, medical treatment as x-rays, network communication as visible light in fiber optics, electrical heaters as infrared, among others. Waves also make instruments such as guitars when a string is plucked. The sound of the piano originates from the resonance when the notes are played. Therefore, the waves have many applications in real life. It is crucial to investigate what causes the sound of various musical instruments. In this case, the exploration seeks to clarify the relationship between resonance and instrument tuning, affecting the sound released. Optimizing the performance of musical instruments through an understanding of the mechanics underlying resonance requires addressing the impact of tension on standing wave frequency.

Background research

Analyzing the forces acting on a string is vital in understanding how tension affects the frequency of a standing wave. When a weight is hanging at the bottom of a string, tension is created, and several forces act on the wire, forming standing waves with different harmonics:

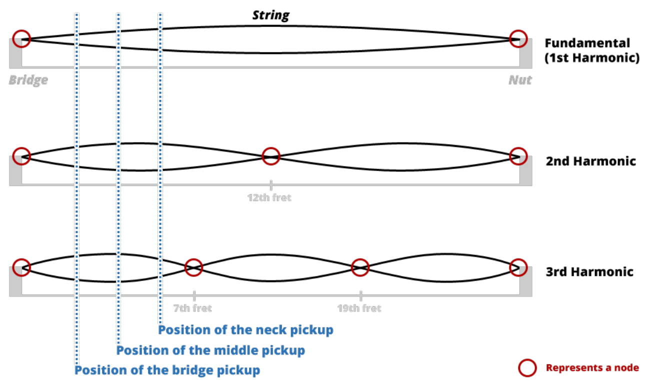


Fig 1: Harmonics (Hassan and LUMS School of Science and Engineering)

Harmonics are the various frequencies at which a wave, such as a sound wave or a string vibration, can naturally oscillate.

Correcting the Uncertainty:

To correct the uncertainty of the mass of the string, we will solve the frequency through the linear density. This will prove whether the value for the frequency is the same as the value found through the second method (which is without the linear density):

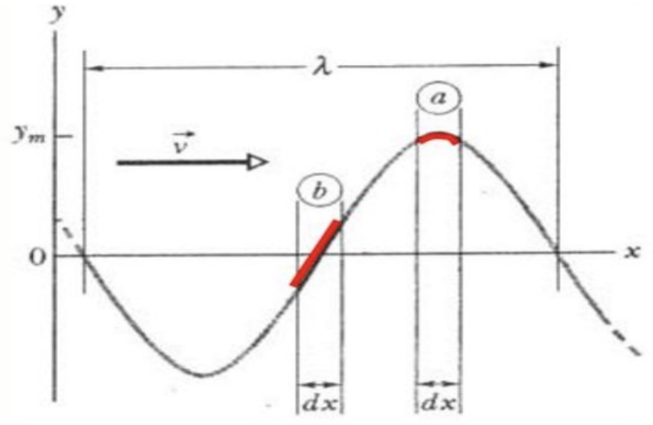
$$f = \frac{1}{2t}\sqrt{\frac{T}{N}}$$

The Formation of Standing Waves:

Standing waves on a string are formed by the superposition of two equal frequency waves traveling in opposite directions, as in a piano where incident waves and their reflections create constant frequency standing waves. The string's tension is a restoring force, pulling the wire back towards equilibrium when displaced. Following Hooke's law, this force is directly proportional to the displacement, which explains the behavior in such scenarios.

The kinetic energy in a standing wave:

The kinetic energy associated with the transverse velocity V_m is associated with a mass dm string element oscillating transversely in SHM when the wave passes through it. The element's transverse velocity reaches its maximum as it races through the y=0 point (element b in Figure 2). The element's kinetic energy and transverse velocity are both zero when it is in its extreme position, y = ym (as is element a).



Formation of wave on a string

The following equation shows the frequency and wavelength of a formed wave;

$\upsilon = \lambda f$,

However, the mass and elasticity of the medium define its qualities, which also dictate how quickly a wave may move through it. On the other hand, these characteristics should allow one to determine the wave's speed across the medium. We make use of the mass of a string element, which is calculated by dividing its mass (m) by its length (l) (Hassan and Anwar). This ratio is known as the string's linear density or μ. Hence, ε = m/l, where m is the mass divided by the length, or $ML{-1}$.

It is crucial to understand that unless a string is under tension—that is, it has been pulled and stretched by forces at both ends—you cannot cause a wave to travel along it. According to Pothuri et al., we can relate the string's elasticity to its tension. Its stretching and tension forces have the same dimension as a force, dF = ma. The following shows the different dimensions of a wave;

- Wave speed υ : LT^{-1}
- Linear density μ: ML^{-1}

- The dimension of the tension and the stretching forces τ: MLT^{-2}

Here, the objective is to combine μ and τ to produce v. This equation can be combined to get

$$V = C\sqrt{\frac{\tau}{\mu}} \quad .$$

When dimensional analysis is unable to establish *C*, a dimensional constant.

Originating from the Second Law of Newton:

Imagine a single, symmetrical pulse traveling at v mph from left to right along a string. It looks like the image below.

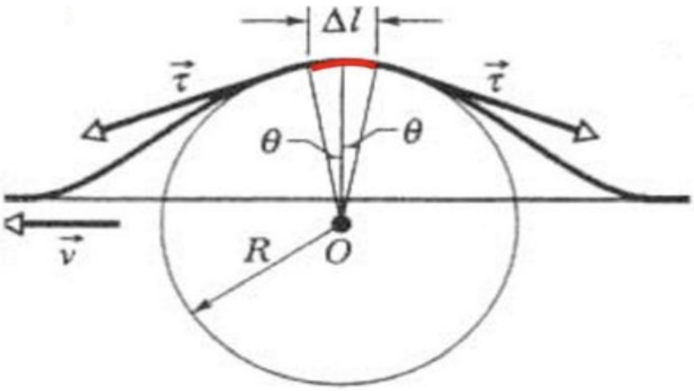


Figure 2: Forces acting on a pulse of a standing wave held at both ends (Freeman)

To make things easier, we pick a reference frame where the pulse doesn't move and run alongside it so it is always visible. The string passes us in this frame, traveling at speed v from right to left. Consider a little string element that subtends a centripetal angle of 2θ and forms an arc of a circle with radius R, with a pulse length of Δ*l* (Adiabatic). This element is pulled tangentially at both ends by a force equal to the string's tension. These forces have a radial restoring force formed by adding their vertical components, whereas their horizontal components cancel.

In magnitude,

$$F(force) = 2(\tau sin0) \approx \tau(\frac{\Delta 1}{R})$$

The mass of the element $\Delta m = \mu\Delta l$

As depicted in Figure 2, the swing element Al is currently traveling in a circle's arc. Consequently, it accelerates centripetally toward the direction of that circle's center, as indicated by;

$$F = \tau(\frac{\Delta 1}{R})$$

$$\Delta m = \mu\Delta l$$

The centripetal acceleration of the arc:

$$a = \frac{v^2}{R}$$

Newton's second law states that $F = ma$,

$$\tau(\frac{\Delta 1}{R}) = (\mu\Delta l)\frac{v^2}{R}$$

Solving the equation for speed gives,

$$v = \sqrt{\frac{\tau}{\mu}}$$

Instead of the wave's frequency, the tension and linear density of the string will determine the wave's speed along it.

Hypothesis:

The hypothesis proposes a proportional relationship between tension and frequency in a standing wave.

Variables:

Independent:

The mass being hung at the end of the string was changed in increments of 100 g. The masses were weighed on the weighing scale to account for potential uncertainty.

Dependent:

The frequency was measured using a frequency generator, which allowed us to induce oscillations in the string at precise frequencies. This was key in observing how changes in tension affected the frequency of the standing waves.

Controlled:

Variable Controlled	**Reason**	**Method**
Length of string	Length directly shapes waves and standing wave formation. We kept a fixed string, securing it at both ends: tied left, tensioned right with weights.	A 2m ruler measured the string's length, which was 1.25m, leaving 25cm for the mass and knot. This length remained constant during the investigation.
Material of String	Consistency mattered. We stuck with polyester string for uniform elasticity, which is crucial due to materials' differing elastic properties affecting tension.	One string was used throughout to address elasticity uncertainty.

The angle between the string and the stand	The angle affects string length. Stands were equal; clamps kept a consistent, horizontal alignment. Any angle change would've altered length, affecting tension and frequency observations.	The stand and vibration generator remained constant, ensuring the angle stayed unchanged.

Methodology:

Apparatus:

Material	Quantity	Uncertainty
Stand	2 – 30 cm tall	N/A
Meter Ruler	1	+-0.01
Frequency Generator	1	+-0.1
String	1	N/A
Pulley	2	N/A
Mass Hanger	1	N/A
Masses	10 x 100g	N/A
Weighing Scale	1	+-0.05 g

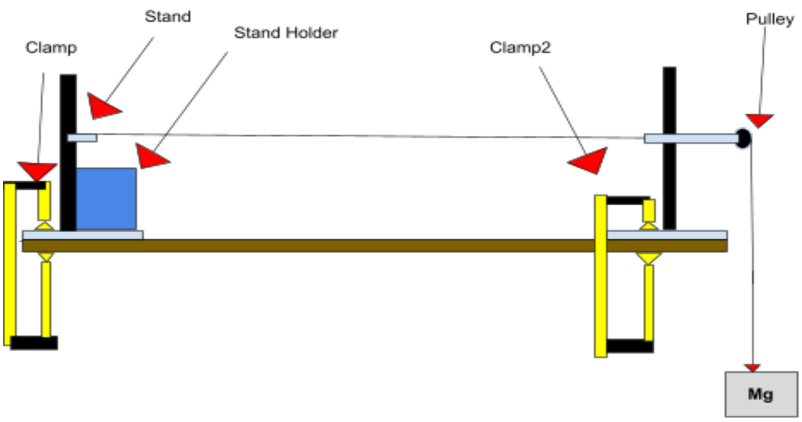


Figure 2: Initial Experiment Setup

Final Methodology:

1. *String Measurement and Preparation:*

The measurement instruments were arranged as shown in the above setup. A measuring rule was used to measure 1.05 cm of the string, and a small loop was created at one end where the mass was attached.

2. *Vibration Generator Setup:*

The string was attached to a vibration generator on the left side. A clamp secured the generator to prevent movement when the mass was added.

3. *Stand and Pulley Alignment:*

The stand was positioned with a pulley at the same height as the generator, ensuring the string remained horizontal and avoiding data inconsistency due to angular displacement. The stand was clamped to prevent tipping under increased weight.

4. *Mass Attachment and Adjustment:*

100g mass was attached at the end of the string, which was used to find the optimal tension without slack. The string had red marks at 1m, leaving 30 cm for the mass attachment.

5. *String Length and Book Adjustment:*

The string's length was gauged at over 1 hour and 30 minutes to facilitate accurate measurement. A thin book was placed under the stand to equalize height, maintaining a consistent string angle relative to the vibration generator.

6. *Frequency and Weight Incrementation:*

A generator's intensity increased the vibration to produce four standing wave nodes. The mass was incrementally increased by 100g, maintaining the node count constant.

7. *Data Recording:*

We recorded the lowest and highest frequencies capable of maintaining four nodes for each weight increment. We repeated this five times per weight and calculated the average frequency for each mass, noting it in a separate column titled 'Averages.'

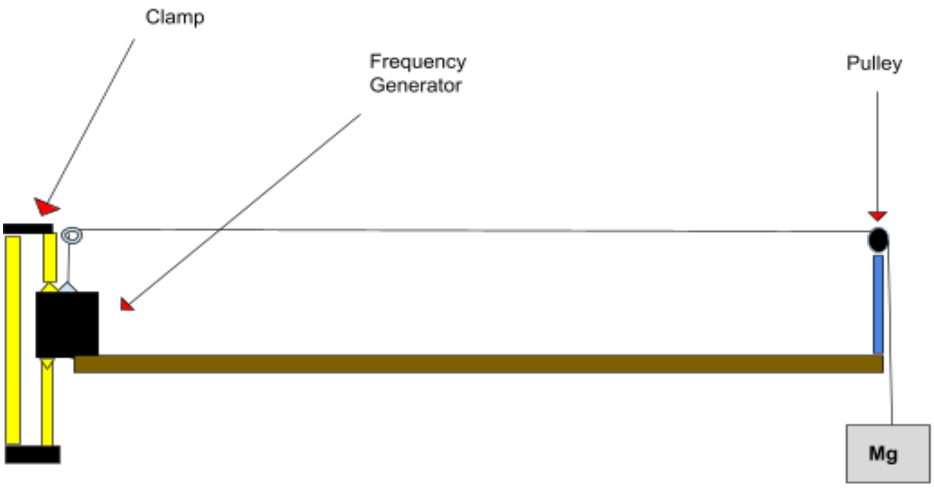


Figure 3: Final Experiment Setup

Safety Considerations:

This experiment, generally safe due to the absence of sharp objects, requires caution when adding mass increment to avoid potential hazards. Placing a pillow beneath the hung mass and storing weights securely is advised. It involves a vibration generator powered by a bank, so ensure dry hands during plug connections to prevent lower skin resistance and dangerous current flow. Protecting liquids from power sources and electrical connections is essential to prevent hazards.

Data Analysis:

The raw data of the experiment is attached in Appendix 1. The tension was calculated in correspondence to each change in frequency to meet four nodes. The uncertainties for the tension were calculated using the mass scale, so our uncertainty is three significant figures for the mass.

Finding the Averages:

The average mass along the three trials can be calculated using:

$$\bar{m} = \frac{1}{3}\sum_{x=1}^{3} m_x$$

Average mass for trial one = $\frac{1}{3}$ x (0. 1001 + 0. 9998 + 0. 1000) = 0.100

The same formula was used to calculate the average frequency needed to form 4 nodes in the three trials,

$$\bar{F} = \frac{1}{3}\sum_{x=1}^{3} F_x$$

Average frequency formed for trial 1 = $\frac{1}{3}$ x (30. 80 + 30. 81 + 30. 79) = 30.80

Processed data:

Mass being hung on the string (±0.001kg)	**Uncertainty in mass (%)**	**Frequency generated**
0.100	0.642	30.80
0.200	0.498	43.55
0.300	0.050	52.63
0.400	0.012	61.00
0.500	0.020	67.37
0.600	0.025	76.20
0.700	0.014	82.67
0.800	0.013	87.37

Table 1: Uncertainty of the Tension.

Calculating the constant, K

The factor variable's theoretical value was ascertained by computing the value of the constant, k. The string's length, density, and frequency values were peculiar to this experiment configuration. The gathered data is then compared to the expected data using this constant.

$k = 4L^2$

The signal generator's frequency was maintained at 30 Hz throughout the experiment. Next, using the meter rule, it was determined that the string length from the vibration generator to the pulley was 1.5 m. The mass per unit length of the string is its density, or u. The string had a mass of 0.006 kg and a total length of 2 meters. The density, calculated by dividing the mass by the length, is roughly 0.003 kg/m^2.

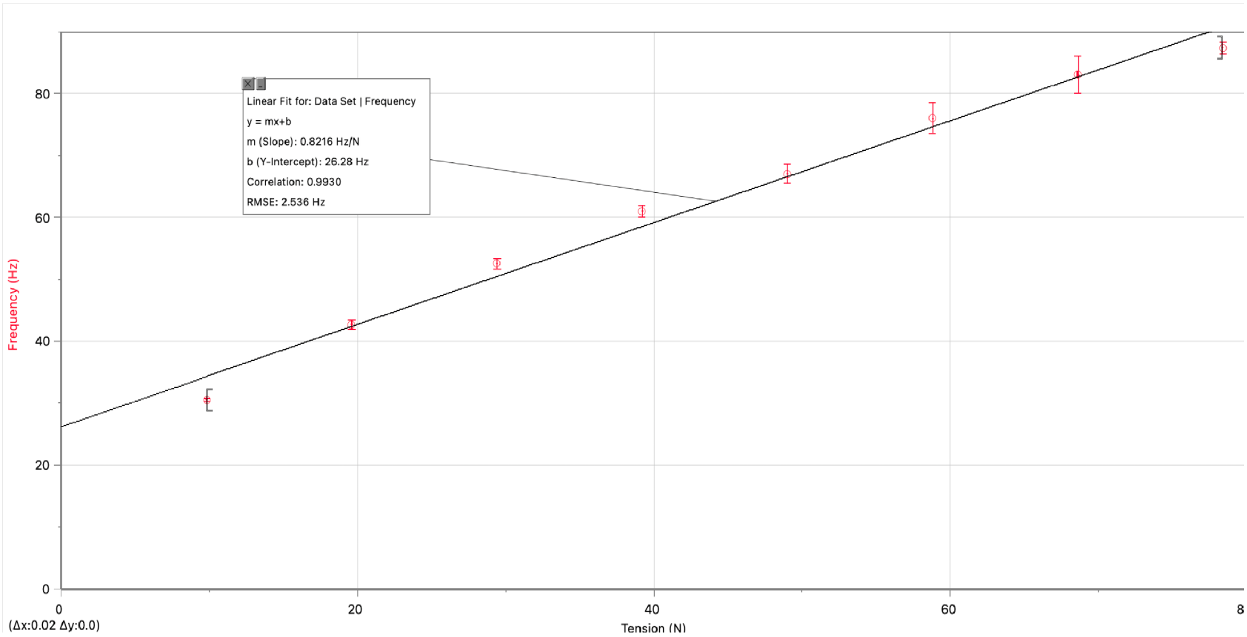
Graphed data

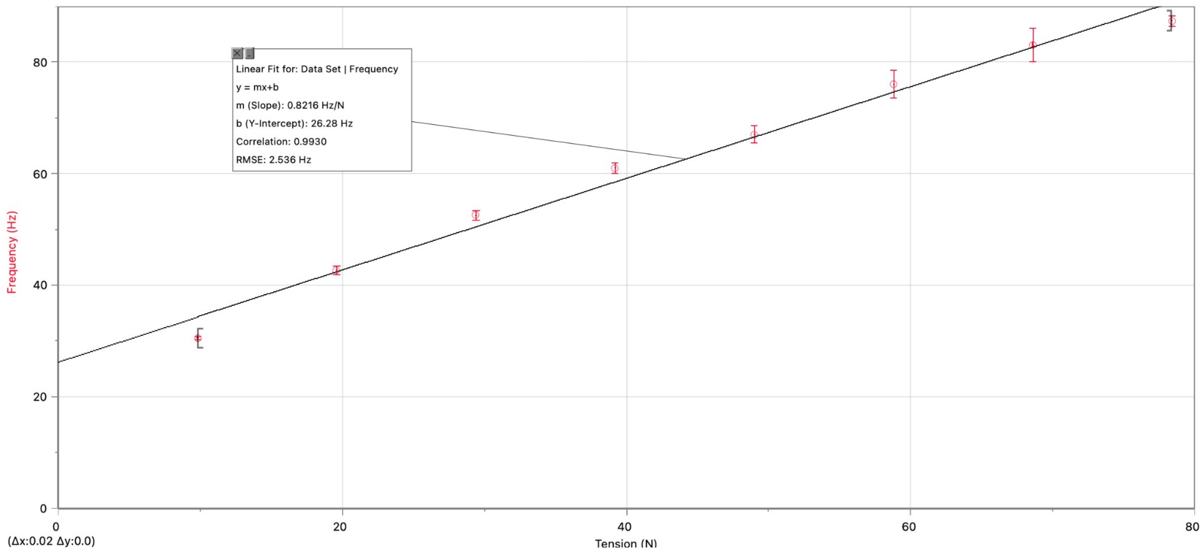
Tension (N)	Uncertainty (%)	Frequency (Hz)	Uncertainty (%)
9.81	0.051	30.50	0.812
19.60	0.027	42.70	1.835
29.40	0.017	52.50	1.495
39.20	0.013	61.00	1.613
49.00	0.012	67.00	2.243
58.80	0.008	76.00	3.335
68.60	0.007	83.00	3.572
78.40	0.006	87.40	1.076

Table 2: Graphed data

Data analysis:

Mass against the number of nodes generated.





All of the gathered data is shown in the graph above. The variables seemed to be positively correlated; when tension rises, so does frequency. The exponential relationship between the mass and number of nodes formed also means that there will be a steep early decrease.

Formulating the theoretical equation:

The predicted line of best fit for this connection, which is exponential, is y = axb. Plotting mass against several produced nodes yields an equation of the line of best fit, which can be derived from equation 2 by substituting the value of K. This is merely a theoretical equation for the particular parameters utilized in the experiment, such as the 1.5 m string length and 30 Hz frequency.

Figure 2: An excellent Logger Pro graph with error bars

The graph plotted above demonstrates the relationship between tension and frequency. Considering that the uncertainties exhibit values below 1%, they could be considered statistically insignificant, creating tiny error bars. The observed relationship between the variables demonstrates linearity, as evidenced by a line of best fit. The trend line R^2 value of 0.997 helped determine the level of correlation. The square root of 0.997 was found to be 0.998, which illustrates a strong association between tension and frequency of harmonics. The equation derived from the trend line was $y = 0.8216x + 26.28$, and it revealed a positive association because there was a positive gradient of 0.8216. Hence, a positive connection was obtained between tension and the square of frequency, suggesting that the increase in tension leads to a higher frequency and, thus, a higher harmonic.

Conclusion

The experiment aimed to understand the properties of a standing wave when the string's tension was changed during the experiment. The results showed a general increase in frequency with rising tension, exemplified by a frequency of 30.50 Hz at a 9.1 N tension. The relationship was also shown on the graph of tension against frequency, where there was an upward trend line. The results indicated that an increase in the string's tension increased the frequency of harmonics. The graph had a trend line with a positive gradient of 0.8216, suggesting a direct relationship.

Further analysis hinted at an inverse linear relationship between tension and the square of the frequency, with a calculated reciprocal of 0.0198 (±0.00004). While affirming the positive correlation between tension and frequency, these findings indicated potential deviations due to experimental uncertainties. The observed pattern, especially the initial plateau, suggests that the relationship might be more accurately represented by a square root function rather than a straightforward exponential one. Therefore, the results confirmed that the change in tension affected the harmonics since the correlation was 0.998.

Evaluation

Strengths:

- **Simplicity:** This experiment was straightforward, using standard equipment and requiring no complex setup or technical expertise. The results were accurate, closely matching the predicted gradient, with minimal uncertainties and manageable effects from a few systematic and random errors, as shown by the slight variations in the gradient extremes.

Limitations and Areas for Enhancement:

- **The presence of friction:** The presence of friction between the pulley and the string that traversed it was observed. Although of minor significance, this shortcoming could have contributed to the occurrence of systematic errors. One potential solution involves employing a pulley constructed from a material that will diminish the amount of friction; such a pulley could be made of LED. Nevertheless, friction will inevitably persist.
- **Oscillations:** The experiment involved adding mass on the hanger during oscillations and introduced random errors.
- **The precision of the weighing scale:** The weighing scale has a significant measurement error of +-0.05 g. In the future, one could reduce the error by measuring mass with a digital scale with little or no measurement error.

Works Cited

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Hassan, Umer, and Muhammad Anwar. *Vibrations on a String and Resonance*. 2015, physlab.org/wp-content/uploads/2016/03/VibrationsV1_march2015.pdf.

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Appendix 1

Table 1: Raw data from the experiment when mass was changed

Mass being hung on the string (± 0.001 kg)			Highest frequency needed to generate Four nodes (Hz)		
Trial 1	Trial 2	Trial 3	Trial 1	Trial 2	Trial 3
0.1001	0.9998	0.1000	30.80	30.81	30.79
0.2020	0.2000	0.2000	43.60	43.63	43.43
0.3001	0.2998	0.3000	53.50	52.50	51.90
0.4001	0.4000	0.4001	62.00	61.00	60.00
0.4998	0.4997	0.4999	69.10	67.00	66.00
0.5999	0.6001	0.5998	77.60	76.00	75.00
0.7001	0.7000	0.6999	84.00	83.00	81.00
0.7998	0.8000	0.8000	88.30	87.40	86.40