Calculating the Volume of a Vase

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Course

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Date

Introduction

In the field of mathematics, characterized by the dominance of numerical and algebraic expressions, it is often the case that unforeseen phenomena give rise to the most remarkable discoveries. During my exploration of the complex terrain of Math Analysis and Approaches, I discovered an unexpected ally in my pursuit of knowledge: a BTPDIAN Vintage porcelain vase. This artifact, steeped in the mysteries of history and mathematics, proved to be a valuable companion on my academic journey.

The vintage porcelain vase, characterized by its intricate patterns and delicate curves, transcended its aesthetic value to become an object of greater significance. The aforementioned object served as a means of access to the process of investigation, facilitating the comprehension of enigmatic mathematical principles. Upon initial observation, the correlation between a porcelain vase and the domain of mathematical equations may be subject to scrutiny. Upon conducting a more thorough examination, the vase divulged an obscured treasure, thereby presenting a challenge that enticed me to

Figure 1: The BTPDIAN Vintage porcelain vase

Rationale

So, what motivated me to select this ostensibly uncomplicated task as a conduit for my mathematical inquiry? The solution can be found in the fundamental nature of mathematics, which involves the exploration of the exceptional within the commonplace. The investigation of volume, a fundamental mathematical notion, is frequently encountered in rudimentary geometric shapes such as cubes or spheres. Through exploring beyond the limitations of traditional academic resources and embracing the surrounding environment, I endeavored to showcase the widespread applicability of mathematical concepts.

Objective

The BTPDIAN Vintage porcelain vase exemplifies the harmonious integration of art and mathematics, where visual appeal seamlessly intersects with mathematical accuracy. Through the analysis of its complex structure, my objective was to utilize mathematical methodologies, including integration, to precisely quantify its volume. This undertaking was not solely a computational exercise, but rather a commemoration of the interdisciplinary character of knowledge, where the refinement of mathematical principles interweaves with the imaginative manifestation of human creativity.

Furthermore, the selection of the vase as the object of investigation was intended to surpass the limitations of theoretical mathematics and establish a link between it and the perceptible realm. The symbol serves as a conduit between the realm of imagination and the physical world, whereby numerical values and mathematical equations are given tangible form. The objective of this undertaking was to stimulate inquisitiveness and admiration for mathematics among individuals who may perceive it as disconnected from their daily routines.

Mathematical Process

I initiated a mathematical exploration aimed at determining the volume of the alluring BTPDIAN Vintage porcelain vase. Equipped with contemporary techniques and a profound comprehension of mathematical concepts, I commence an expedition that amalgamates aesthetics and science, interlacing the complex designs of the vase with the refined equations of mathematics. The aforementioned procedure comprises multiple stages, including the creation of a graph representing the vase, partitioning it into distinct segments, determining the functions that define said segments, and computing its volumetric measurement.

Step 1: Graphing the Vase

The initial phase of this investigation entails utilizing Desmos, a robust graphical computational tool that will facilitate the visualization of the morphology of the vase. The graphical representation is obtained through a meticulous process of plotting individual data points and subsequently connecting them with fluid curves, thereby exposing the fundamental nature of the vase's structure (Desmos, 2023). This was possible using the measurements obtained from Google regarding this particular vase.

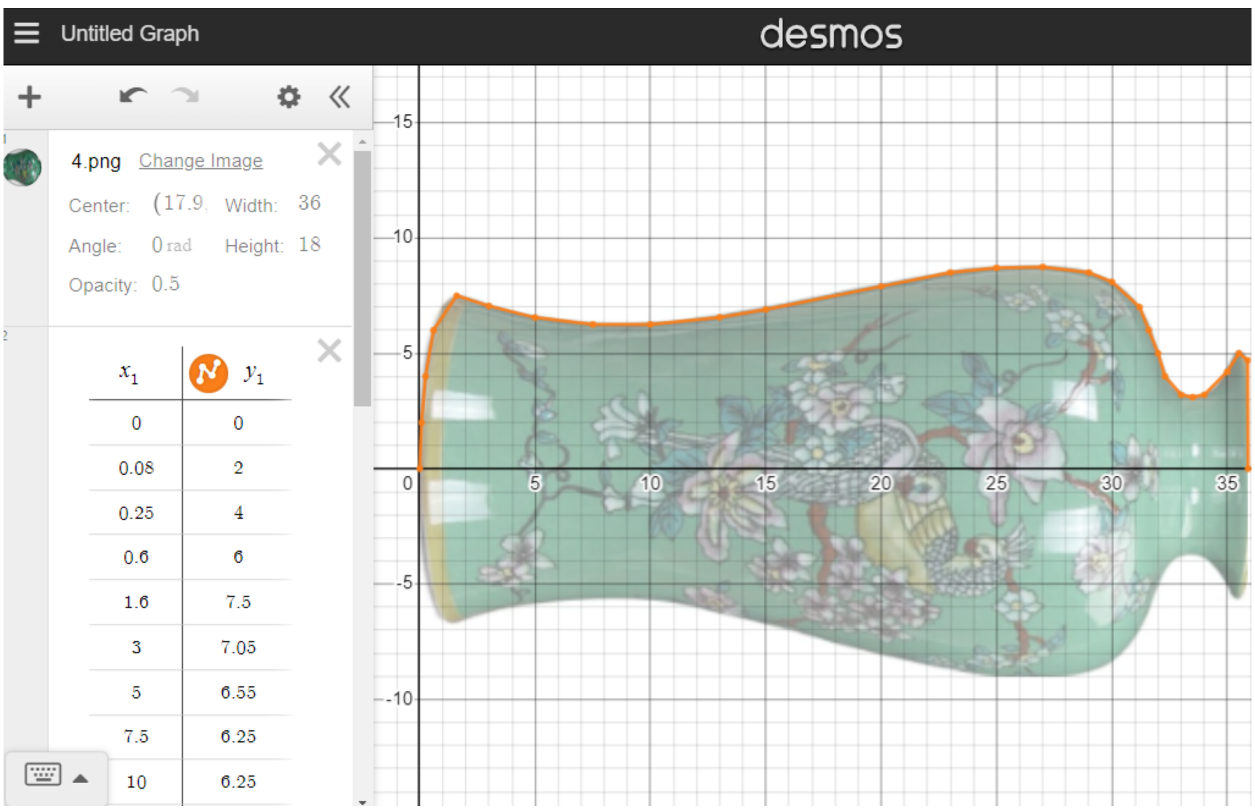

Figure 2: Indicating the graphing of the vase in desmos

Through the collaborative dance of mathematics and technology, the intricate contours of the vintage porcelain masterpiece come to life on the screen, awaiting further scrutiny.

Step 2: Piecewise Functions

As I delve deeper, the complexity of the vase's shape becomes apparent. It possesses varying dimensions, curves, and sections, each demanding meticulous attention. To capture its essence mathematically, piecewise functions enter the stage. The piecewise function in every section indicated in Figure 3 below will be generated using the langrage interpolation.

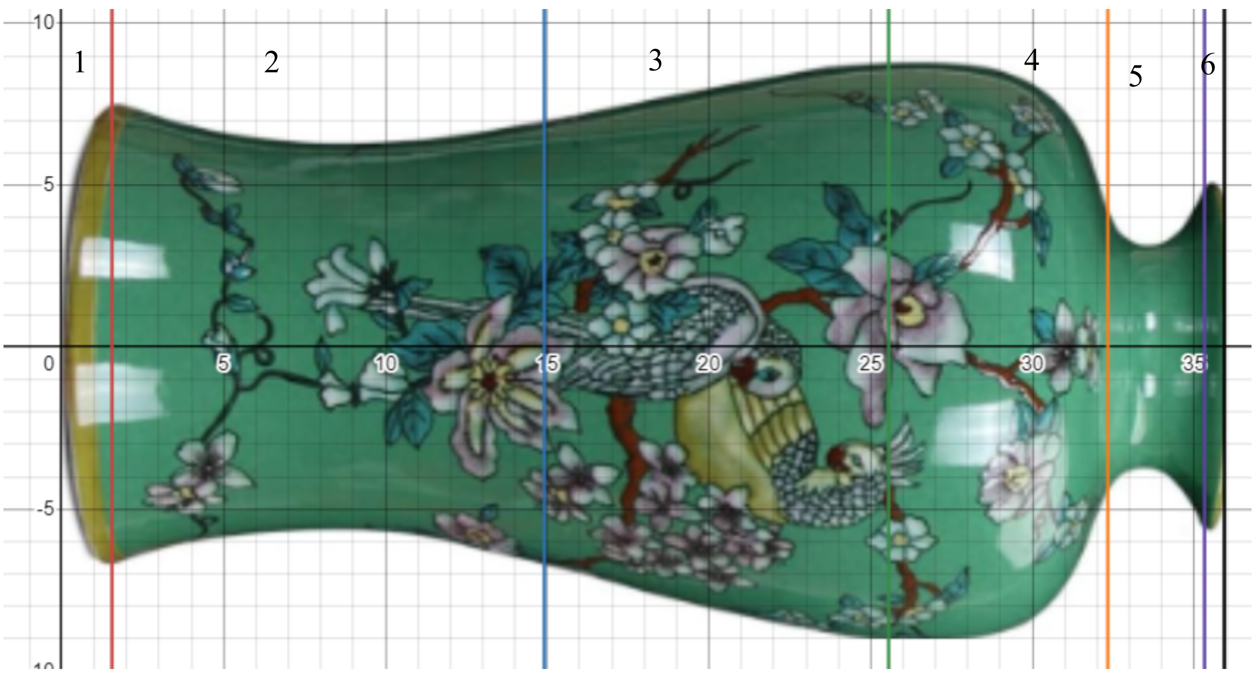

Figure 3: indicating the six identified sections

Section 1: Quantic Function

After carefully analyzing this section, I discovered that a quantic function describes it best. The below table of values was generated from Figure 2.

Table 1:

Points	1	2	3	4	5	6	7

x	0	0.094	0.245	0.45	0.6	1.14	1.545
y	1.73	2.800	4.08	5.204	6.009	7.2	7.514

Since I do not need all the points, I will select five points from Table 1 for interpolation. I will use points 1, 2, 3, 4, and 7. I will have five Lagrange basis polynomials in this case since we have five data points. Let's denote them as $L_1(x)$, $L_2(x)$, $L_3(x)$, $L_4(x)$, and $L_5(x)$ for the four data points. From the langrage theorem, the langrage basis polynomial can be computed using the below formula. Suppose we have three points $\left(x_0,\ x_1\right),\ \left(x_1,\ y_1\right),\ and\ \left(x_2,\ y_2\right)$ (Simplilearn, 2022). Then, the langrage basis polynomial can be generated as shown below.

$$L_1(x) = \frac{y_0\left(x - x_1\right)\left(x - x_2\right)}{\left(x_0 - x_1\right)\left(x_0 - x_2\right)},\ l_2(x) = \frac{y_1\left(x - x_0\right)\left(x - x_2\right)}{\left(x_1 - x_0\right)\left(x_1 - x_2\right)}\ldots\ldots$$

$$L_1(x) = \frac{1.73(x - 0.094)\left(x - 0.245\right)\left(x - 0.45\right)\left(x - 1.545\right)}{(0 - 0.094)\left(0 - 0.245\right)\left(0 - 0.45\right)\left(0 - 1.545\right)}$$

$$L_1(x) = 108.04661555x^4 - 252.1808007x^3 + 150.68018935x^2 - 30.42966535x + 1.73$$

$$l_2(x) = \frac{2.8(x - 0)\left(x - 0.245\right)\left(x - 0.45\right)\left(x - 1.545\right)}{(0.094 - 0)\left(0.094 - 0.245\right)\left(0.094 - 0.45\right)\left(0.094 - 1.545\right)}$$

$$l_2(x) = -\ 381.88783135x^4 + 855.42874223x^3 - 452.16473951x^2 + 65.04934111x$$

$$l_3(x) = \frac{4.08(x - 0)(x - 0.094)\left(x - 0.45\right)\left(x - 1.545\right)}{\left(0.245 - 0\right)\left(0.245 - 0.094\right)\left(0.245 - 0.45\right)\left(0.245 - 1.545\right)}$$

$$l_3(x) = 413.82804379x^4 - 864.48678349x^3 + 365.3191205x^2 - 27.04511106x$$

$$l_4(x) = \frac{5.204(x-0)(x-0.094)\left(x-0.245\right)\left(x-1.545\right)}{\left(0.45-0\right)\left(0.45-0.094\right)\left(0.45-0.245\right)\left(0.45-1.545\right)}$$

$$l_4(x) = -144.71274978x^4 + 272.63882058x^3 - 79.12676088x^2 + 5.14907499x$$

$$l_5(x) = \frac{7.514(x-0)(x-0.094)\left(x-0.245\right)\left(x-0.45\right)}{\left(1.545-0\right)\left(1.545-0.094\right)\left(1.545-0.245\right)\left(1.545-0.45\right)}$$

$$l_5(x) = 2.3546037x^4 - 1.85778232x^3 + 0.41342131x^2 - 0.02440193x$$

Now let us conduct the summation of the langrage basis polynomial to get the polynomial function as follows:

$$f(x) = L_1(x) + L_2(x) + L_3(x) + L_4(x) + L_5(x)$$

$$\frac{\left\{\begin{matrix} 108.04661555x^4 - 252.1808007x^3 + 150.68018935x^2 - 30.42966535x + 1.73 \\ -381.88783135x^4 + 855.42874223x^3 - 452.16473951x^2 + 65.04934111x \\ 413.82804379x^4 - 864.48678349x^3 + 365.3191205x^2 - 27.04511106x \\ -144.71274978x^4 + 272.63882058x^3 - 79.12676088x^2 + 5.14907499x \\ 2.3546037x^4 - 1.85778232x^3 + 0.41342131x^2 - 0.02440193x \end{matrix}\right\} +}{-2.37131809x^4 + 9.5421963x^3 - 14.87876923x^2 + 12.69923776x + 1.73}$$

The final function modeling this part of the wine glass is given as follows

$$y = -2.371x^4 + 9.542x^3 - 14.879x^2 + 12.699x + 1.73$$

Graphing this function produces the following

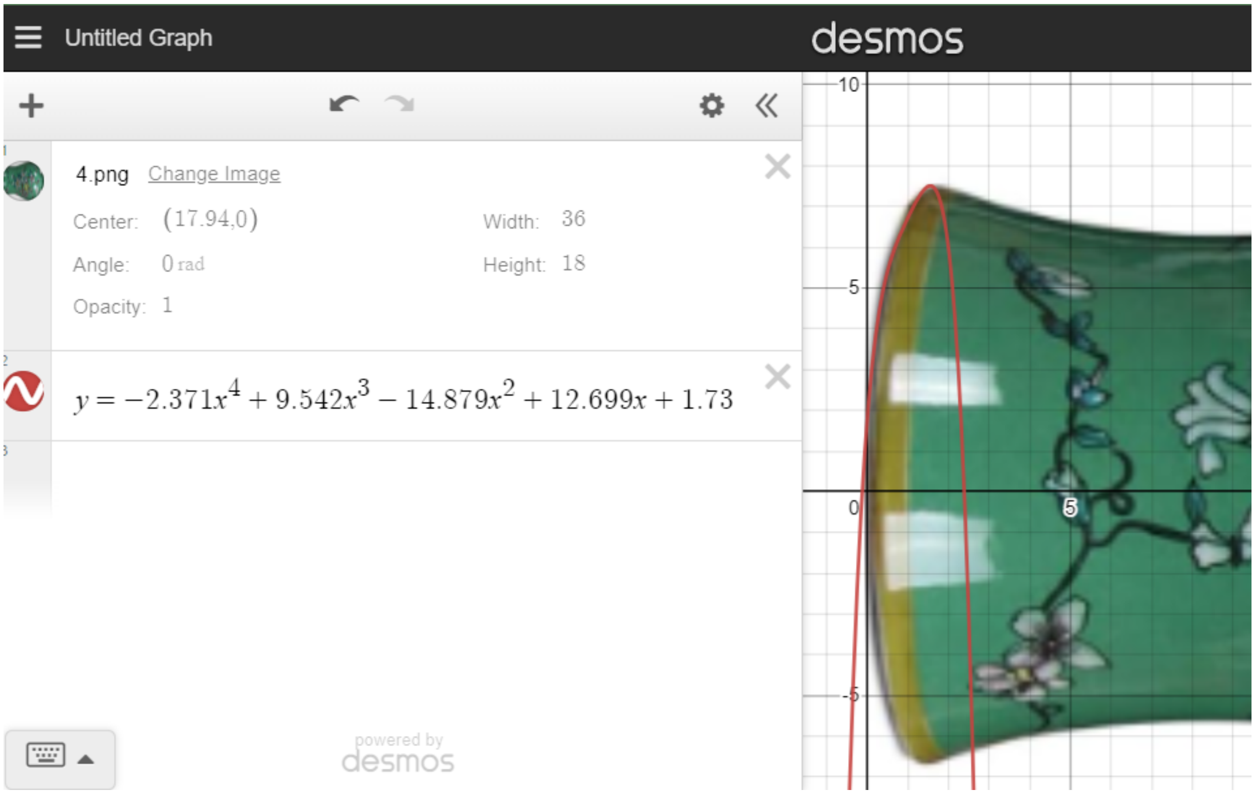

Figure 5: Indicating the modeled first section

Section 2: Quadratic Function

Again on a detailed analysis and viewing of the second section, I found out that a quadratic function best modeled it. In this case, only three points are required per the langrage interpolation. The points I found best to generate the relevant function are shown in Table 2 below.

Points	1	2	3
x	1.448	6	14.908
y	7.467	6.39	6.976

Calculation

$$L_1(x) = \frac{7.467\left(x-6\right)(x-14.908)}{\left(1.448-6\right)(1.448-14.908)}$$

$$L_1(x) = 0.12187056x^2 - 2.54806985x + 10.90107863$$

$$l_2(x) = \frac{6.39(x-1.448)(x-14.908)}{\left(6-1.448\right)\left(6-14.908\right)}$$

$$l_2(x) = -\,0.15758627x^2 + 2.57748115x - 3.40178093$$

$$l_3(x) = \frac{6.976(x-1.448)\left(x-6\right)}{(14.908-1.448)\left(14.908-6\right)}$$

$$l_3(x) = 0.058181x^2 - 0.43333211x + 0.50547655$$

Now let us conduct the summation of the langrage basis polynomial to get the polynomial function as follows:

$$f(x) = L_1(x) + L_2(x) + L_3(x) + L_4(x) + L_5(x)$$

$$\frac{\left\{\begin{matrix} 0.12187056x^2 - 2.54806985x + 10.90107863 \\ -0.15758627x^2 + 2.57748115x - 3.40178093 \\ 0.058181x^2 - 0.43333211x + 0.50547655 \end{matrix}\right\}+}{0.02246529x^2 - 0.40392081x + 8.00477425}$$

In this case, the final approximated function is shown below

$$y = 0.022x^2 - 0.404x + 8.00$$

Graphing this function generated the below image

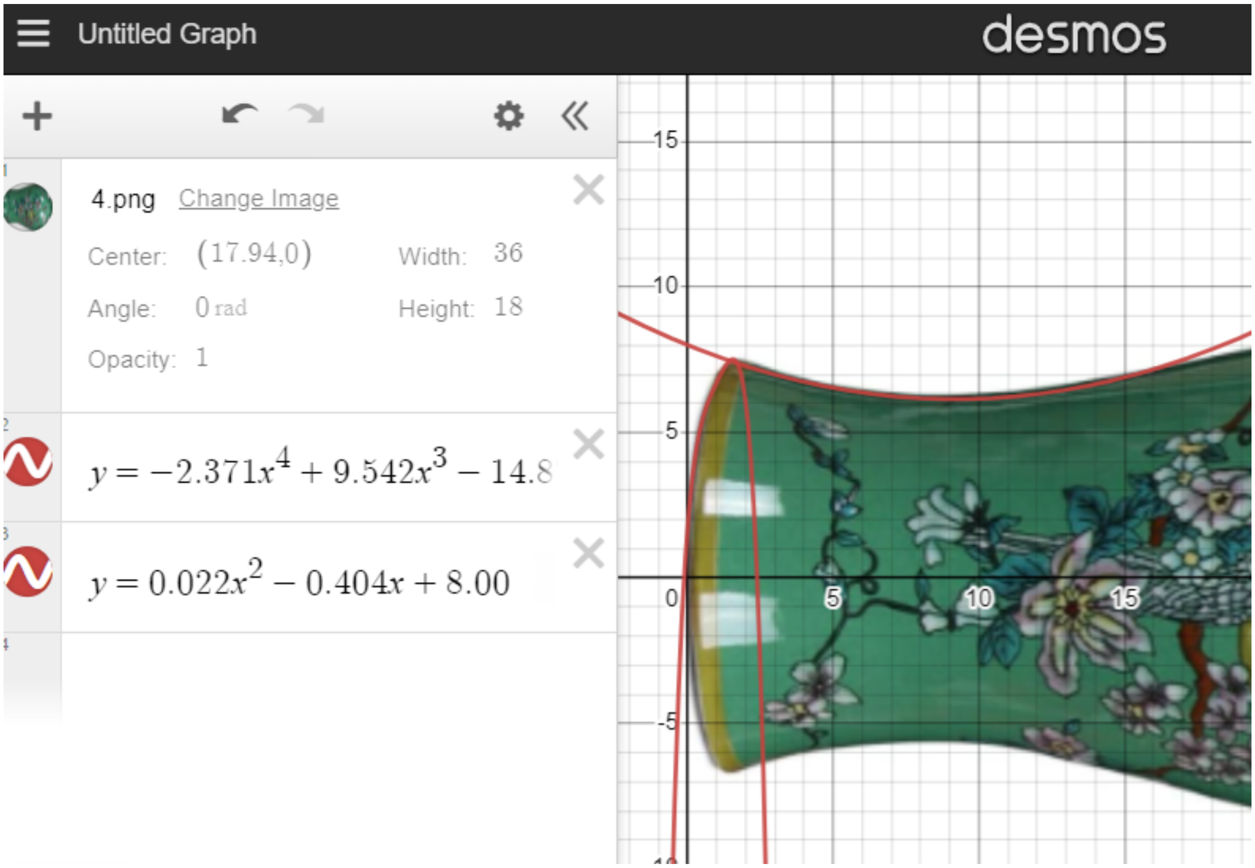

Figure 6: Indicating the modeled section 2

Section 4: Cubic Function

Observing this section of the vase, I observed a little difference from the other sections since I figured a cubic function should model it. From the figure, I generated the below table of values

Table 3: Indicating the values that were used to generate the cubic function for this section

x	27.428	28.732	30.036	32.644
y	8.833	8.606	8.065	3.558

Interpolation

$$L_1(x) = \frac{8.833(x-28.732)\left(x-30.036\right)(x-32.644)}{(27.428-28.732)\left(27.428-30.036\right)(27.428-32.644)}$$

$$L_1(x) = -0.49794969x^3 + 45.51857736x^2 - 1385.00571434x + 14028.03341988$$

$$l_2(x) = \frac{8.606(x-27.428)\left(x-30.036\right)(x-32.644)}{(28.732-27.428)\left(28.732-30.036\right)(28.732-32.644)}$$

$$l_2(x) = 1.29374091x^3 - 116.5764063x^2 + 3492.68934876x - 34792.6027496$$

$$l_3(x) = \frac{8.065(x-27.428)(x-28.732)(x-32.644)}{\left(30.036-27.428\right)\left(30.036-28.732\right)(30.036-32.644)}$$

$$l_3(x) = -0.90930924x^3 + 80.75029811x^2 - 2383.6162741x + 23392.41036094$$

$$l_4(x) = \frac{3.558(x-27.428)(x-28.732)\left(x-30.036\right)}{\left(32.644-27.428\right)\left(32.644-28.732\right)\left(32.644-30.036\right)}$$

$$l_5(x) = 0.06685931x^3 - 5.76300558x^2 + 165.46898776x - 1582.57398397$$

Now let us conduct the summation of the langrage basis polynomial to get the polynomial function as follows:

$$f(x) = L_1(x) + L_2(x) + L_3(x) + L_4(x) + L_5(x)$$

$$\frac{\left\{\begin{matrix} -0.49794969x^3 + 45.51857736x^2 - 1385.00571434x + 14028.03341988 \\ 1.29374091x^3 - 116.5764063x^2 + 3492.68934876x - 34792.6027496 \\ -0.90930924x^3 + 80.75029811x^2 - 2383.6162741x + 23392.41036094 \\ 0.06685931x^3 - 5.76300558x^2 + 165.46898776x - 1582.57398397 \end{matrix}\right\}+}{-0.0467x^3 + 3.9331x^2 - 110.57x + 1046.3}$$

In this case, the final approximated function is shown below

$$y = -0.0467x^3 + 3.9331x^2 - 110.57x + 1046.3$$

Graphing this function generated the below image

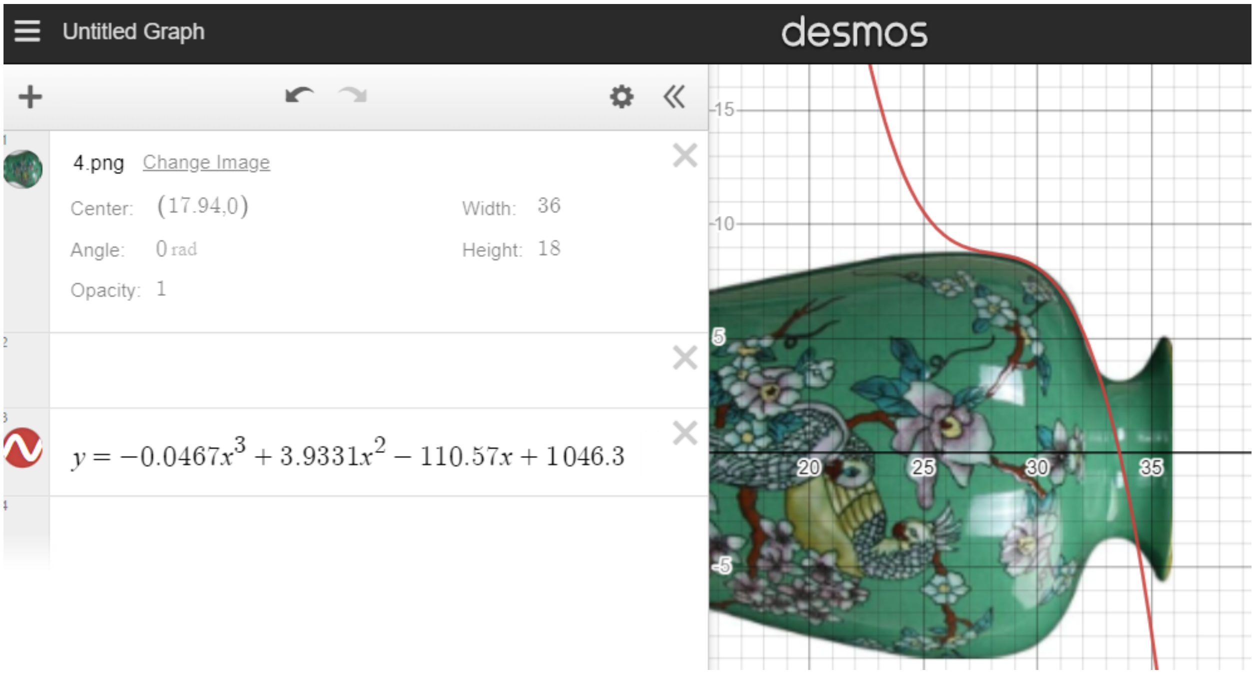

Figure 5: Indicating the generated cubic function

Sections 3, 5 and 6

In this section, a similar finding was identified as in section indicating that they were best modeled using quadratic functions. Therefore, I conducted a similar process by obtaining the respective tables of values, which allowed me to conduct a langrage interpolation that produced the below three functions.

$$3 \rightarrow y = -0.005x^2 + 0.36x + 2.72$$

$$5 \rightarrow y = 0.671x^2 - 45x + 757.5$$

$$6 \rightarrow y = -x^2 + 71x - 1255.25$$

Below is a complete diagram indicating all the generated piecewise functions in one graph

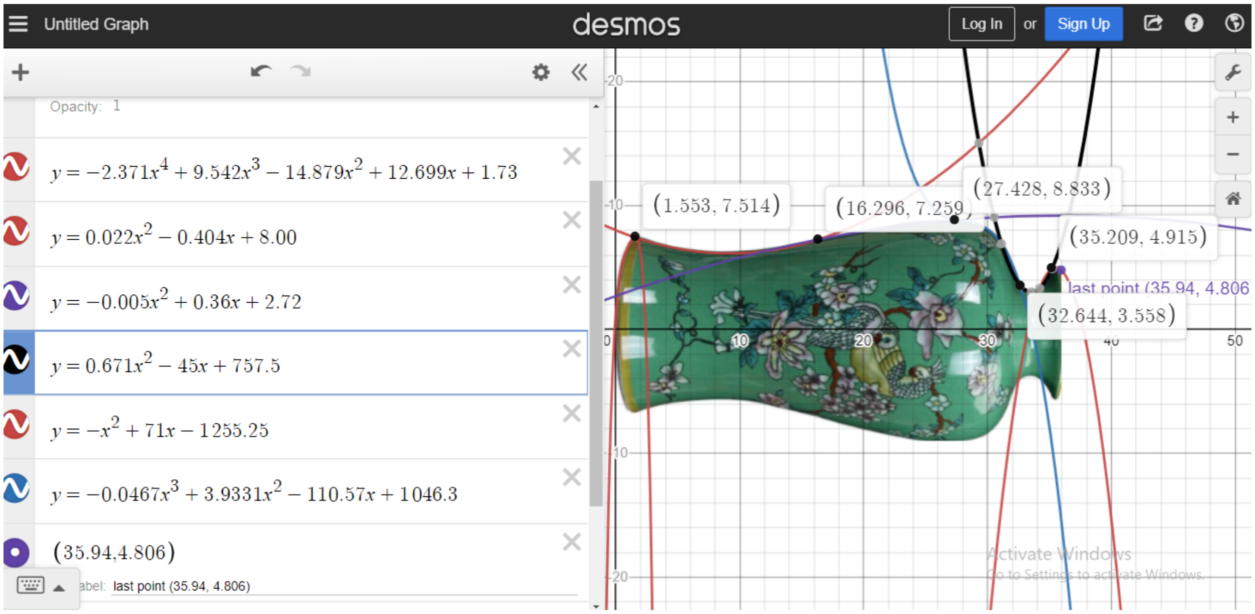

Figure 6: Indicating all the generated piecewise function and their respective integrals

A comprehensive representation of its form is obtained by dividing the vase into distinct sections, each with its own mathematical expression. Piece by piece, these functions stitch together to create a complete mathematical tapestry, mirroring the intricacy of the vase itself.

Step 3: The Volume of the Vase

The true revelation lies in the calculation of volume. To accomplish this, the volume of the revolution formula takes center stage. By rotating the graph of the vase around a chosen axis, a three-dimensional shape is generated, encapsulating the very essence of the porcelain masterpiece (Paul’s, 2023). The formula's elegant simplicity becomes a key to unlocking the secrets of the vase's volume, transforming it into a mathematical abstraction waiting to be discovered.

With the graph in hand (figure 6) and armed with the volume of the revolution formula, the final act of this mathematical spectacle begins. Integrating the appropriate functions, each corresponding to a distinct section of the vase, an amalgamation of infinitesimal slices fills the stage. Through the integration process, the collective sum of these slices unravels the volume, revealing the hidden numerical treasure contained within the porcelain vessel, as shown below.

The Formula of Volume of Revolution

The volume of revolution is a method used in calculus to find the volume of a solid obtained by rotating a curve around an axis. In this case, the vase was rotated along the x-axis; therefore, the formula below will be used (brilliant.org, 2023).

$$v = \int_a^b \pi\left(f(x)\right)^2 dx$$

Section 1: Volume

$$y = -2.371x^4 + 9.542x^3 - 14.879x^2 + 12.699x + 1.73$$

$$integrals \rightarrow a = 0\ b = 1.553$$

$$v = \pi\int_0^{1.553} \left(-2.371x^4 + 9.542x^3 - 14.879x^2 + 12.699x + 1.7\right)^2 dx$$

$$= \pi\int_0^{1.553} 5.621641x^8 - 45.248164x^7 + 161.605982x^6 - 344.169494x^5 + 455.670957x^4 - 345.454042x^3 + 110.676001x^2 + 43.1766x + 2.89$$

$$\pi\left[\frac{5.621641x^9}{9} - \frac{45.248164x^8}{8} + \frac{161.605982x^7}{7} - \frac{344.169494x^6}{6} + \frac{455.670957x^5}{5} - \frac{345.454042x^4}{4} + \frac{110.676001x^3}{3} + \frac{43.1766x^2}{2} + 2.89x\right]_0^{1.553}$$

$$\boldsymbol{v} \approx 55.346\boldsymbol{\pi}$$

Section 2: Volume

$$y = 0.022x^2 - 0.404x + 8$$

$$integrals \rightarrow a = 1.553\ and\ b = 16.296$$

$$v = \int_{1.553}^{16.296} \pi\left(0.022x^2 - 0.404x + 8\right)^2 dx$$

$$= \pi \int_{1.553}^{16.296} 0.000484x^4 - 0.017776x^3 + 0.515216x^2 - 6.464x + 64dx$$

$$= \pi \left[\frac{0.000484x^5}{5} - \frac{0.017776x^4}{4} + \frac{0.515216x^3}{3} - \frac{6.464x^2}{2} + 64x \right]_{1.553}^{16.296}$$

$$\boldsymbol{v} \approx 633.495\boldsymbol{\pi}$$

Section 3: Volume

$$y = -0.005x^2 + 0.36x + 2.72$$

$$integrals \rightarrow a = 16.296 \; and \; b = 27.428$$

$$v = \int_{16.296}^{27.428} \pi\left(-0.005x^2 + 0.36x + 2.72\right)^2 dx$$

$$= \pi \int_{16.296}^{27.428} 0.000025x^4 - 0.0036x^3 + 0.1024x^2 + 1.9584x + 7.3984dx$$

$$= \pi \left[\frac{0.000025x^5}{5} - \frac{0.0036x^4}{4} + \frac{0.1024x^3}{3} + \frac{1.9584x^2}{2} + 7.3984x \right]_{16.296}^{27.428}$$

$$\boldsymbol{v} \approx 741.546\boldsymbol{\pi}$$

Section 4: Volume

$$\mathrm{y} = -0.0467\mathrm{x}^3 + 3.9331\mathrm{x}^2 - 110.57\mathrm{x} + 1046.3$$

$$integrals \rightarrow a = 27.428 \; and \; b = 32.644$$

$$v = \int_{27.428}^{32.644} \pi\left(-0.0467\mathrm{x}^3 + 3.9331\mathrm{x}^2 - 110.57\mathrm{x} + 1046.3\right)^2 dx$$

$$= \pi \int_{27.428}^{32.644} \left(0.00218089x^6 - 0.36735154x^5 + 25.79651361x^4 - 967.490154x^3 + 20456.12996x^2 - 231378.782x + 1094743.69\right) dx$$

$$= \pi \left[\frac{0.00218089x^7}{7} - \frac{0.36735154x^6}{6} + \frac{25.79651361x^5}{5} - \frac{967.490154x^4}{4} + \frac{20456.12996x^3}{3} - \frac{231378.782x^2}{2} + 1094743.69x \right]_{27.428}^{32.644}$$

$$\boldsymbol{v} \approx 299.830\boldsymbol{\pi}$$

Section 5: Volume

$$y = 0.671x^2 - 45x + 757.5$$

$$integrals \rightarrow a = 32.644 \; and \; b = 35.209$$

$$v = \int_{32.644}^{35.209} \pi\left(0.671x^2 - 45x + 757.5\right)^2 dx$$

$$= \pi \int_{32.644}^{35.209} (0.450241x^4 - 60.39x^3 + 3041.565x^2 - 68175x + 573806.25) dx$$

$$= \pi \left[\frac{0.450241x^5}{5} - \frac{60.39x^4}{4} + \frac{3041.565x^3}{3} - \frac{68175x^2}{2} + 573806.25x \right]_{32.644}^{35.209}$$

$$\boldsymbol{v} \approx 32.117\boldsymbol{\pi}$$

Section 6: Volume

$$y = -x^2 + 71x - 1255.25$$

$$integrals \rightarrow a = 35.209 \; and \; b = 35.94$$

$$v = \int_{35.209}^{35.94} \pi\left(-x^2 + 71x - 1255.25\right)^2 dx$$

$$= \pi \int_{35.209}^{35.94} (x^4 - 142x^3 + 7551.5x^2 - 178245.5x + 1575652.5625) dx$$

$$= \pi \left[\frac{x^6}{5} - \frac{142x^4}{4} + \frac{7551.5x^3}{3} - \frac{178245.5x^2}{2} + 1575652.5625x \right]_{35.209}^{35.94}$$

$$\boldsymbol{v} \approx 17.912\boldsymbol{\pi}$$

Final Result

The total volume of the vase is equivalent to the summation of the volumes for the six sections, as shown below:

$$v_T \approx 55.346\pi + 633.495\pi + 741.546\pi + 299.830\pi + 32.117\pi + 17.912\pi \approx 1780.246\boldsymbol{\pi}\, \boldsymbol{cm}^3$$

As the calculations unfold, the numbers intertwine with the vase's beauty, bridging the worlds of aesthetics and mathematics. The final result, a precise numerical value, represents the culmination of this mathematical odyssey—a testament to the power of exploration and the ability of mathematics to illuminate the world around us.

Limitation

This exploration was not without limitations. For instance, one limitation of the exploration is the potential inaccuracy in generating the piecewise functions. Piecewise functions can be complex and require careful consideration of the various segments and their equations. Nevertheless, owing to the intrinsic subjectivity entailed in formulating these functions, the possibility of errors or inaccuracies in their construction cannot be discounted.

An additional constraint pertains to the unavailability of pre-computed volume data for the relevant vase, thereby impeding comparative analysis. The availability of pre-existing data or benchmarks can be advantageous for comparison and evaluation when investigating a specific topic or problem (Friedrich & Friede, 2023). Nonetheless, in this investigation, there may be a shortage of pre-determined volumes that can be utilized as benchmarks. The lack of comparative data may impede the evaluation of the precision, productivity, or efficacy of the produced piecewise functions (Friedrich & Friede, 2023). Therefore, the assessment of the degree to which the exploration has effectively approximated or modeled the intended volumes may pose a challenge.

Conclusion

To conclude, the BTPDIAN Vintage porcelain vase served as a source of inspiration for me, motivating me to further investigate the intricacies of mathematics and uncover its concealed dimensions. The convergence of art and science was exemplified by the experience, which served as a reminder that mathematical exploration is not limited to traditional educational settings, but rather permeates the diverse fabric of the surrounding environment. As I initiated the process of calculating its volume, I embarked on an exploratory expedition within myself and the extensive realm of mathematical prospects. By utilizing Desmos, piecewise functions, and the volume of revolution formula, this mathematical exploration elevates the BTPDIAN Vintage porcelain vase from a mere object of aesthetic appreciation to a vessel of mathematical inquiry. The techniques of graphing, function analysis, and integration are employed to uncover the mysteries of its volume, thereby establishing a seamless interconnection between the domains of art and science. This expedition illuminates the quantitative assessment of the vase and enhances our comprehension of the intricate relationship between mathematics and the aesthetic qualities inherent in the artifacts that encompass our environment.

Work Cited

brilliant.org. (2023, accessed). *Volume of Revolution | Brilliant Math & Science Wiki*. https://brilliant.org/wiki/volume-of-revolution/

Desmos. (2023, accessed). *Desmos | Graphing Calculator*. Desmos. https://www.desmos.com/calculator

Friedrich, S., & Friede, T. (2023). On the role of benchmarking data sets and simulations in method comparison studies. *Biometrical Journal*, *n/a*(n/a), 2200212. https://doi.org/10.1002/bimj.202200212

Paul's. (2023, accessed). *Calculus I - Volumes of Solids of Revolution / Method of Rings*. https://tutorial.math.lamar.edu/classes/calci/volumewithrings.aspx

Simplilearn. (2022, December 8). *What is Lagrange Interpolation? An Overview | Simplilearn*. Simplilearn.Com. https://www.simplilearn.com/tutorials/statistics-tutorial/lagrange-interpolation