# Mathematics Internal Assessment

# Modeling a soccer penalty shot

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**Introduction**

From a very tender age, I have closely followed different world football competitions, including the FiFa World Cup, Copa America, and Nation league. Besides being a fan, I have also participated in football in school and during the holidays. Out of the several special plays I admire, I rank penalty kicks at a precisely higher position, a factor that has made me admire notable penalty kickers, including Frank Lampard, Alan Shearer, Cristiano Ronaldo, and Zinedine Zidane, among others. I have keenly observed how they take and score penalty kicks at different points. Behind all this, in mathematics class, I have been taught different concepts, including functions and geometry, and their applications in real-life. Thus, I have thought it wise to combine my mathematical knowledge of functions and geometry and my long admiration for penalty kicks into designing a mathematical investigation on modeling a soccer penalty kick.

**The Plan of the Investigation**

This investigation is aimed at modeling a soccer penalty kick. To achieve this, a tape measure will be used to accurately measure a distance of 12 meters from the goal position. After that, I will be assisted by my IB student to record the trajectory of my penalty kick with the aid of a stopwatch and a professional camera. The data obtained will be used to model the penalty kick using the knowledge of functions and geometry. The height of the standard goal used in this investigation was 2.48 m, and the ball's trajectory will be assumed to be reaching the maximum height before going past the goal position and striking the net.

**Important Data, Measured and Collected During the Penalty Kick**

The Height of the Goal: 2.48 m

The vertical distance by which the ball did hit the net: was 2.41 m

The ball takes the total time from the penalty spot to the position of the net: 2.15 seconds

**Mathematical Process**

The assumed trajectory of the penalty kick with the geometrical measurements can be presented in figure 1 below.

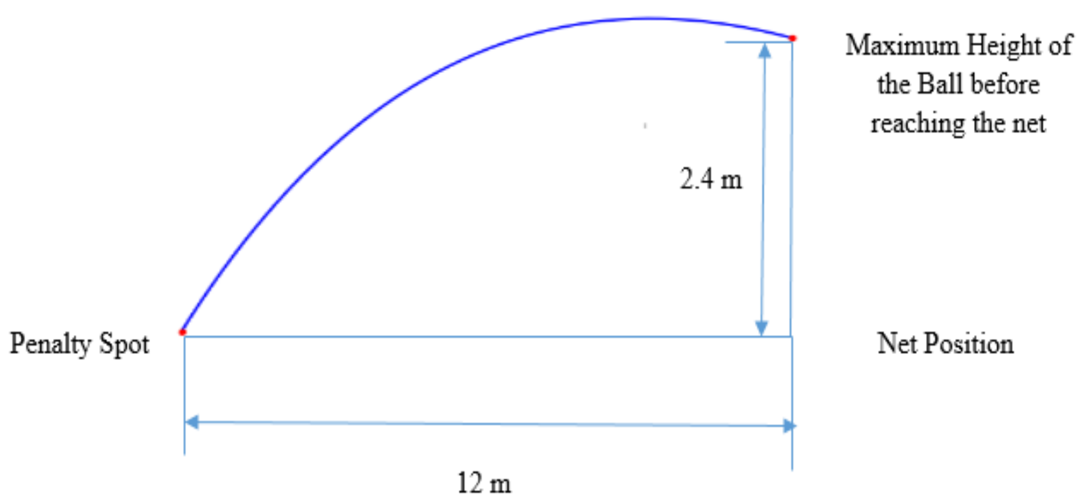


*Figure 1: Geometrical Representation of the Penalty Trajectory*

In modeling the trajectory of the penalty kick designed above, a general function of assumed vertical Displacement by the curve in figure 1 above will be modeled using the knowledge of functions. The mathematical model will make use of the SUVAT equations to determine the maximum point reached by the ball, the vertical velocity, the horizontal velocity, and the kick angle.

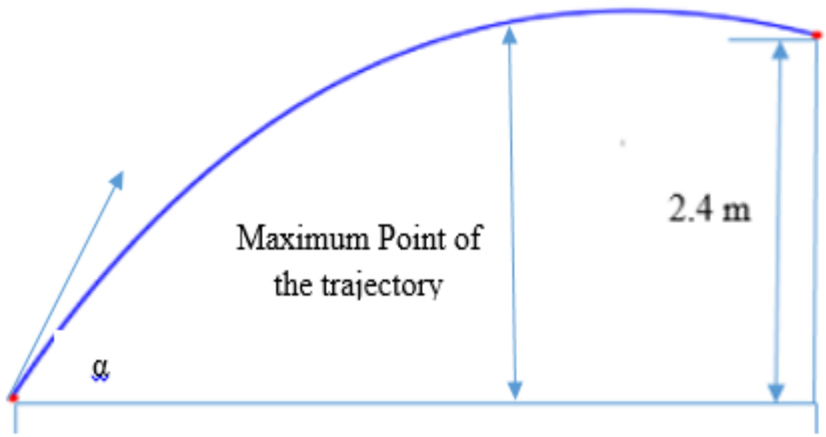


*Figure 2: Illustration for the Maximum Point on the Trajectory and the Kick Angle*

In accordance with the study of NCL (1), the Displacement of any object can be determined through the following equation;

$$\text{Displacement, } S = ut + \frac{1}{2}at^2$$

Where;

$u =$ The initial velocity of the object

$t =$ The total time taken by the object in the Displacement

$a =$ The acceleration of the object

With respect to this mathematical investigation, the penalty distance of 12 m will be taken as the Displacement, S. The time taken by the ball towards the net, 2.15 seconds, will be taken as the value of t, and v as the resultant velocity of the ball. When kicked, the initial velocity of the soccer ball would be split into two to have a vertical and a horizontal component based on the kick angle defined in figure 2 above.

$$\text{The initial horizontal component of the velocity of the soccer ball, } u_H = u \cos\alpha$$

$$\text{The initial vertical component of the velocity of the soccer ball, } u_V = u \sin\alpha$$

With these initial and vertical velocity components of the soccer ball, the initial velocity, u, can be considered to be the hypotenuse in a right-angled triangle and computed through a Pythagoras geometrical formula;

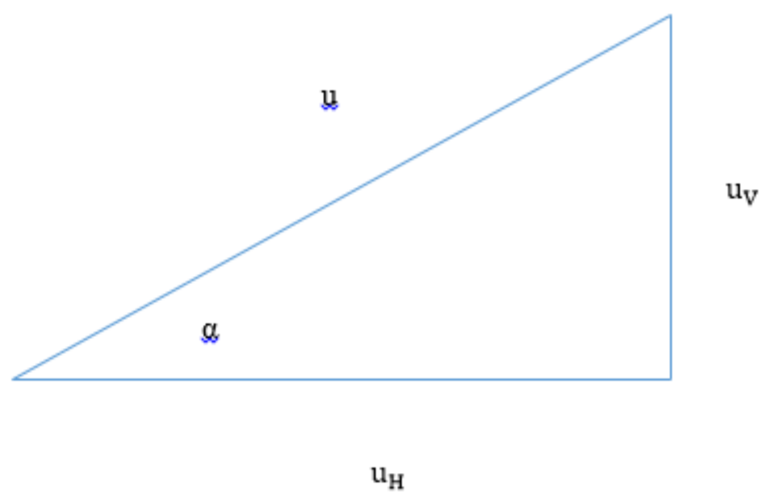


*Figure 3: Representation of the Initial Velocity of the Penalty kick on a Right-Angled Triangle*

In addition to the right-angled triangle in figure 3 above, the horizontal displacement component of the soccer ball could be de determined defined from the general SUVAT equation defined above;

$$\text{Horizontal Displacement, } S_x = ut + \frac{1}{2}at^2$$

With the substitution of the horizontal component of the initial velocity;

$$\text{Horizontal Displacement, } S_x = \text{ucos}\alpha\text{t} + \frac{1}{2}\text{at}^2$$

Considering that the initial velocity of the soccer ball will be constant throughout the motion of the ball, the acceleration would be taken as zero. Thus;

$$\text{Horizontal Displacement, } S_x = \text{ucos}\alpha\text{t} + \frac{1}{2}(0)\text{t}^2$$

$$S_x = \text{ucos}\alpha\text{t}$$

With a substitution of 2.15 seconds and 12 m distance into this equation, the horizontal distance of the ball would be;

$$12 = \text{ucos}\alpha\ (2.15)$$

$$12 = 2.15\text{ucos}\alpha$$

The vertical displacement component of the ball could be determined in a similar way, only changing the velocity component;

$$\text{Vertica Displacement, } S_y = \text{usin}\alpha\text{t} + \frac{1}{2}a\text{t}^2$$

In line with the study of Gaur (1), when an object is undergoing a vertical displacement, its acceleration is usually the gravitational acceleration that acts in the opposite direction. Thus, the vertical displacement equation would be;

$$\text{Vertica Displacement, } S_y = \text{usin}\alpha\text{t} - \frac{1}{2}\text{gt}^2$$

With the substitution of 2.15 seconds, 2.40 m distance, and 9.81 m/s² as the acceleration due to gravity into the vertical displacement equation of the soccer ball;

$$\text{Vertica Displacement, } 2.40 = \text{usin}\alpha\,(2.15) - \frac{1}{2}(9.81)(2.15)^2$$

$$2.40 + \frac{1}{2}(9.81)(2.15)^2 = 2.15\text{usin}\alpha$$

$$25.073 = 2.15\text{usin}\alpha$$

The horizontal and the vertical displacements equations were then solved simultaneously to find the values of the unknown;

$$12 = 2.15\text{ucos}\alpha$$

$$25.073 = 2.15\text{usin}\alpha$$

From the first equation;

$$\text{u} = \frac{12}{2.15\cos\alpha} = \frac{5.581}{\cos\alpha}$$

Substituting into the second equation;

$$25.073 = 2.15\left(\frac{5.581}{\cos\alpha}\right)\sin\alpha$$

$$25.073 = 12\ \tan\alpha$$

Thus, it would be possible to compute the exact value of the angle the penalty had been kicked through;

$$\tan\alpha = \frac{25.073}{12} = 2.0894$$

$$\alpha = \tan^{-1}(2.0894) = 64.42°$$

The corresponding initial velocity of the soccer ball will be;

$$u = \frac{5.581}{\cos\alpha} = \frac{5.581}{\cos(64.42°)} = 12.93\ m/s$$

Having obtained the initial velocity and the angle of a penalty kick, it would be possible to work out the maximum height reached by the ball. The maximum height or position of the ball is usually reached when the final velocity is zero. The final velocity of the soccer ball kick in a vertical projection can be determined through a SUVAT equation. In line with the study of Study.Com (2), the general SUVAT equation for velocity is usually;

$$\text{Final velocity,}\ v = u\sin\alpha - gt$$

Substituting the value of v as 0 m/s, and g as 9.81 $ms^{-2}$ into this equation;

$$0 = 12.93\ \sin 64.42 - 9.81t$$

$$9.81t = 12.93\ \sin 64.42$$

$$t = \frac{12.93\ \sin 64.42}{9.81}$$

$$= 1.189\ \text{seconds}$$

The time value obtained can be used to work out the vertical distance reached (maximum vertical Displacement) through the general equation that had been utilized in the preceding steps of this subjection;

$$\text{Maximum Vertica Displacement, } H = u\sin\alpha t - \frac{1}{2}gt^2$$

$$= 12.93 \sin 64.42\ (1.189) - \frac{1}{2}(9.81)(1.189)^2$$

$$= 13.867 - 6.934$$

$$= 6.933 \text{ m}$$

At this point, the horizontal Displacement of the ball could be determined using the general equation:

$$\text{Horizontal Displacement, } S_x = u\cos\alpha t + \frac{1}{2}(0)t^2$$

$$= 12.93 \cos 64.42\ (1.189)$$

$$= 6.638 \text{ m}$$

Thus, the maximum distance of the soccer ball in a penalty kick will be reached at a distance slightly more than half of the 12 m distance. Of more importance, however, for this investigation, the assumed trajectory of the ball in the penalty kick at any time can be modeled through the general equation;

$$\text{The trajectory of the ball at any instance of time, } S_y = 12.93\sin 64.42\ t - \frac{1}{2}(9.81)t^2$$

$$S_y = 11.662\ t - 4.905\ t^2$$

To represent this model graphically, a table of values of time was created, as illustrated in table 1 below;

*Table 1: Finding the Values of Displacement of the Soccer ball Trajectory at different times*

| Time, t (seconds) | The trajectory of the ball, $S_y$ (Meters) |
|---|---|
| 0.000 | 0.000 |
| 0.400 | 3.880 |
| 0.800 | 6.190 |
| 1.189 | 6.932 |
| 1.600 | 6.102 |
| 2.000 | 3.704 |
| 2.150 | 2.400 |

The trajectory of the soccer ball was then developed using the Microsoft Excel application.

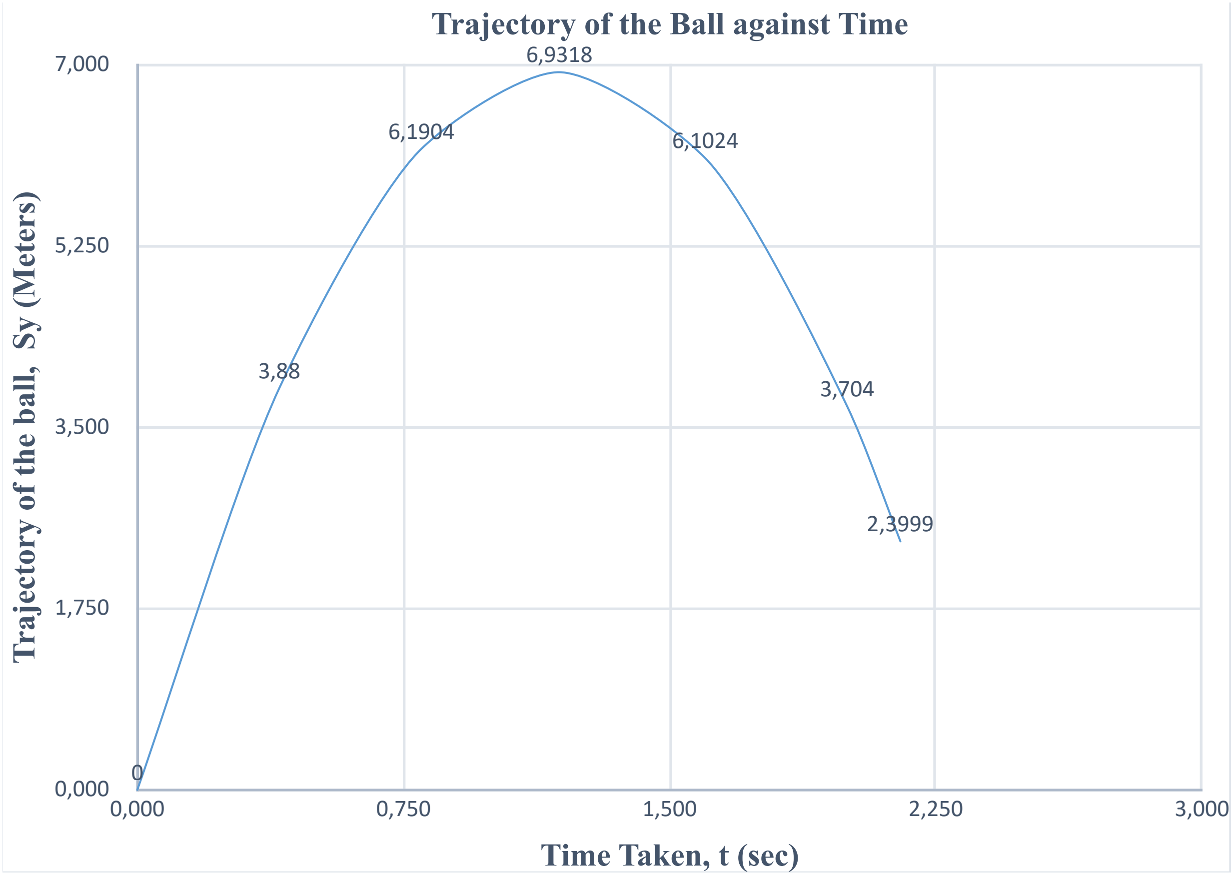


*Figure 4: A Graph Representing the Trajectory of the Ball against Time*

The curve in figure 4 above shows that the soccer ball, when kicked, rises from 0.000 m distance to 6.932 m progressively, which starts to drop, reaching the net after 2.150 seconds and at a vertical distance of 2.400m.

**<u>Conclusion and Evaluation</u>**

This mathematics investigative study was aimed at modeling a soccer penalty kick. A tape measure was first used to accurately measure a distance of 12 meters from the goal position. After that, the time the ball took to go past the goal and hit the net was measured with the aid of the stopwatch. These pieces of data were processed through SUVAT equations with the application of geometry concepts to model the general function for the trajectory of the soccer ball in the penalty kick. The processed data revealed that the maximum distance reached by the soccer ball during the penalty kick is 6.932 m, and the general function for the trajectory of the penalty kick is $S_y = 11.662t - 4.905t^2$. The trajectory obtained in this investigation showed that the ball, when kicked, rises from 0.000 m distance to 6.932 m progressively, which starts to drop, reaching the net after 2.150 seconds. The investigation was the decision and the possibility of finding the angle of the penalty kick. This angle was found to be very instrumental in modeling the penalty kick. However, there were some notable limitations in developing the investigation. First, the use of only one trial of time did not reveal the true picture of the personal ability of the penalty kick since it might not have been accurate. Instead, it would have been important to take at least five trials of time and average them for more accurate results. Besides this, any errors made while tracking the time with the stopwatch may have affected the accuracy of the model of the trajectory. In the future, this error can be avoided by increasing the number of time measurements and using other means of recording time, such as the iPhone application or a loggerPro, to increase the level of accuracy. Furthermore, only one mathematical approach was used in this investigation. It would have been better to use at least two different mathematical models for the soccer ball shot and compare them in terms of strengths and weaknesses to find the most applicable.